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CENTRALIZED RESOURCE ALLOCATION IN TWO-STAGE PRODUCTION SYSTEMS: A NETWORK DEA APPROACH

Abstract. *Network data envelopment analysis (NDEA) is recently developed to explore the internal structure of network production systems so that the efficiencies are measured more precisely. Centralized Resource Allocation (CRA) is a method in which all DMUs are projected onto the efficiency frontier through solving just one DEA mode. This paper proposed a centralized network data envelopment analysis model that combines the centralized data envelopment analysis model and network data envelopment analysis to allocate resources among sub-units. The novel proposed non-radial centralized NDEA approach provides improvement of all inputs and outputs in a unified model. A simple numerical example is presented to illustrate the applicability of the approach. An empirical application on the 8 Chinese commercial banks is also provided for illustration and analysis.*

***Keyword**: Data envelopment analysis; Centralized resource allocation; Two-stage production; Efficiency.*

JEL Classification: C61, C67

1. Introduction

Data envelopment analysis (DEA) is a useful mathematical programming tool for evaluating performance of Decision Making Units (DMUs) that can contribute to economic and productivity growth in which each DMU consumes multiple inputs to produce multiple outputs. In the basic radial models introduced by Charnes, et al. (1978) and Banker et al. (1984), in analyzing the relative performance of the units, each DMU is separately projected onto the efficient frontier and the percentage of reduction (decreasing) of the inputs (outputs) that can be attained. DEA is extensively used in measuring and analyzing performance of homogenous production units in many different sectors like education, health care, finance, utilities, transportation, etc., see for example Kazemi Matin, et al. (2007) as an application of imprecise DEA in education. Emrouznejad and Yang (2018) provide an extensive list of DEA applications from 1978 to end of 2016.

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Recent DEA developments exhibit an additional planning orientation for the resource allocation problem. The use of DEA models provides an alternative way to the resource allocation problem and allows to consider feasible production plans and trade-offs between the inputs and outputs Korhonen and Syrjänen (2004). However, situations may occur where a single decision-maker controls all DMUs. The centralized decision-maker might mostly consider enhancing the efficiency of an entire organization, rather than increasing the performance of each DMU separately. Hence, a centralized decision-maker aims to allocate resources to optimize the operations of all DMUs globally.

In recent years, a large number of authors have developed DEA models for resource allocation in a centralized environment. In this framework, Golany et al. (1993) suggested an output model for the resource allocation axis, which used high inputs for the total inputs and as a result, allocated resources. Athanassopoulos (1995) and Athanassopoulos (1998) presented a DEA-based Goal Programming Data Envelopment Analysis (GODEA) model, as well as a multiplier form-based programming model for centralized planning. Lozano and Villa (2004) implemented a resource allocation DEA model, by focusing on the minimization of the total input consumed by all the DMUs and presenting a linear programming model, in which all units are assumed to be on the efficiency boundary. Lotfi et al. (2010) proposed a Centralized Data Envelopment Analysis (CDEA) model based on the enhanced Russell measure, which allowed all DMUs to be easily projected onto the efficient frontier through solving only one model. Fang (2013) attempted to control all decision-making units by a central unit, through combining the technical efficiency and attribute efficiency components. In order to obtain the combined efficiency of the two components, they applied the structural efficiency to further decompose it into three components of the aggregate technical efficiency, aggregate allocative efficiency, and re-transferable efficiency components. In order to improve the environmental performance of the units under evaluation and to maximize their satisfaction, Wu et al. (2018) presented a model for identifying the maximum income of the evaluated units in the resources reallocation process based on the best income.

In recent years, various studies have been also carried out on the two-stage systems – systems that produce multiple outputs by consuming inputs in the first stage and then, use them as the inputs for the second stage to produce final outputs. In this regard, some of these studies have used modified classical DEA models for the two-stage systems. Färe and Grosskopf (1996) and Seiford and Zhu (1999) studied the network DEA in the former type of decomposition, and found some mathematical relationship between the overall efficiency and the component efficiencies, although no specific relationship was observed between those two components in the latter type. Lewis and Sexton (2004) used the same method to study the performance of Major League. Kao and Hwang (2008) measured the total system efficiency by calculating the efficiency of sub-systems with precise middle sizes and using relational models. Considering the structure of the data envelopment analysis of two-stage data, each decision-maker has two successive stages of production. Cook, Zhu et al. (2010) developed data envelopment analysis models with a network structure and introduced a multi-stage model, in which the outputs of each stage can be considered as the final product and exit the system or enter into the next stage as inputs. In addition, new inputs can be logged in at each step in Classic closed systems. However, it is not possible to enter new inputs and the output of the final stage is known as the system's final output. In this case, research has addressed the breakdown of the technical efficiency between the sub-sectors of production and scaling to obtain the overall system efficiency, with regard to the efficiency of the sub-sectors Kao and Hwang (2011). By using the analysis of the banking industry data and applying the SBM model, Zhu et al. (2018) represented the logic of the leading and advanced ideas presented for two-stage production networks and developed a model to study these production systems. Li et al. (2019) suggested a min-max model and its probabilistic algorithm, in order to guarantee a unique allocation program by repeatedly reducing the deviation for all units. Ahranjani et al. (2018) This research aims to develop a two-stage network DEA model to study the economic notion of economies of scope (ES) between two products.

Chen, Yu et al. (2018) recommended a model that could serve as a guide to resource allocation in shipping lines in the presence of undesirable outputs. This paper reviewed the involved factors and offered a two-stage network CDEA (NCDEA) model, by integrating the two-stage NDEA and CDEA of resource allocation for internal lines of a shipping company in Taiwan in 2013.

In this research, we investigated the unique mode of allocation in a two-stage network. aiming to evaluate a general mode of resource allocation in two-stage production networks. This study mainly focuses on Lozano and Villa's method (Lozano and Villa (2004), which is a two-stage centralized resource allocation, to decrease the total inputs or increase the total outputs. In the following, a single-stage model is proposed to reduce the input and increase the output simultaneously based on the idea of the previous paper Lotfi et al. (2010). Ultimately, both of these articles were reviewed in single-stage production networks (black boxes).

The main purpose of this research is to focus on resource allocation in the two-stage production networks and in the presence of intermediate products. For this purpose, we examined the constraints imposed by the intermediate products and presented a model with numerical and empirical examples to consider these issues.

The rest of the paper is organized as follows. Section 2 presents a brief review of the classic DEA model and the centralized resource allocation in DEA framework. Section 3 provides an extension of the CDEA model for evaluating the performance of a two-stage network system. Section 4 illustrates the model by providing a simple numerical example. Section 5 explains the application of the two-stage network DEA for evaluating commercial banks in Chinese, in addition to comparing the results. Finally, Section 6 concludes the research and provides some suggestions for future research.

2. Preliminaries

In DEA, it is most common to characterize each observed DMU_j as follows: j= 1,…,n represent the indexes for observed DMUs; i=1,…,m indicates the index for inputs; k=1,…,s shows the index for outputs; x_{ij} demonstrates the amount of input *I* consumed by DMU j; y_{kj} displays the quantity of output k produced by DMU j; θ and φ are the radial contraction of the total input vector and the total output vector, respectively, and $\lambda_r = (\lambda_{1r}, \lambda_{2r}, \ldots, \lambda_{nr})$ is the vector of intensity weights for producing DMU_r.

The BCC model is regarded as one of the most popular basic DEA models introduced by (Banker, Charnes et al. 1984) to measure the efficiency of the whole production system (black-box). The input-oriented BCC model could be present as:

$$\begin{aligned} & \min \quad \theta \\ & s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \qquad \forall i \\ & \qquad \sum_{j=1}^{n} \lambda_j y_{kj} \ge y_{ko} \qquad \forall k \\ & \qquad \lambda_j \ge 0 \qquad \forall j \\ & \qquad \sum_{j=1}^{n} \lambda_j = 1 \end{aligned} \tag{1}$$

The dual of the above linear programming problem is as follows:

$$\begin{aligned} & \max \quad \sum_{k=1}^{s} u_k y_{ko} - u_o \\ & s.t. \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \qquad \sum_{k=1}^{s} u_k y_{kj} - u_o - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad \forall j \\ & \qquad u_k, v_i \ge 0 \end{aligned} \tag{2}$$

Lozano and Villa (2004) proposed a centralized BCC DEA model for setting separate targets for each DMU in one development. The model assumes that there is a centralized decision-maker (DM), which aims to maximize the efficiency of individual units while minimizing the total input consumption or maximizing the total output production. Their input-oriented radial model is stated as follows:

$$
\begin{aligned}
& \min \quad \theta \\
& s.t \quad \sum_{r=1}^{n} \sum_{j=1}^{n} \lambda_{jr} x_{ij} \le \theta \sum_{j=1}^{n} x_{ij} \qquad \forall i \\
& \qquad \sum_{r=1}^{n} \sum_{j=1}^{n} \lambda_{jr} y_{kj} \ge \sum_{j=1}^{n} y_{kr} \qquad \forall k \\
& \qquad \sum_{j=1}^{n} \lambda_{jr} = 1 \quad \forall r \\
& \qquad \lambda_{jr} \ge 0
\end{aligned}
\tag{3}
$$

This model jointly projects each of the observed DMUs onto the Pareto efficiency frontier. However, the model is defined in a black-box production system. The next section extends the centralized resource allocation for the case of two-stage production system with intermediate produce.

3. Using Centralized Resource Allocation in network DEA: New Approach

Figure 1 illustrates a typical two-stage system, in which z_{gj} (g=1,…,h) represent the outputs of the first stage that are consumed as inputs for the second stage (intermediate products).

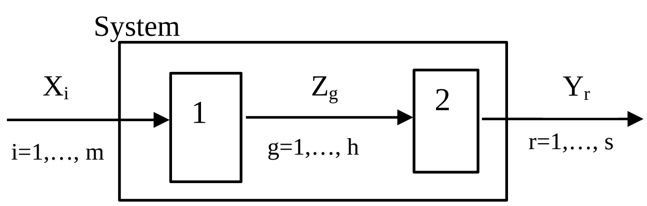

Figure 1. Simple two stage network

The following model is proposed by Kao and Hwang (2008) for system efficiency evaluation of two-stage production in multiplier form, by considering stages constraints:

$$
\begin{aligned}
\max \quad & \sum_{k=1}^{s} u_k y_{ko} \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{k=1}^{s} u_k y_{kj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j \\
& \sum_{g=1}^{h} w_g z_{gj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j \\
& \sum_{k=1}^{s} u_k y_{kj} - \sum_{k=1}^{h} w_g z_{gj} \le 0 \quad \forall j \\
& u_k, w_g, v_i \ge 0
\end{aligned} \tag{4}
$$

Taking the dual, gives the following envelopment form of the model (4):

$$
\begin{aligned}
\min \quad & \theta \\
s.t \quad & \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{io} \quad \forall i \\
& \sum_{j=1}^{n} \lambda_j^1 z_{gj} \ge \sum_{j=1}^{n} \lambda_j^2 z_{gj} \quad \forall g \\
& \sum_{j=1}^{n} \lambda_j^2 y_{kj} \ge y_{ko} \quad \forall k \\
& \lambda_{jr}^1 \ge 0 \quad \forall r, j \\
& \lambda_{jr}^2 \ge 0 \quad \forall r, j
\end{aligned} \tag{5}
$$

Now, we introduce a new centralized resource allocation for performance of two-stage production systems. A Russell type efficiency evaluation technique is proposed in the new centralized model to better discriminate the observed units.

Consider the following two-stage network DEA model:

$$\text{Min} \quad \gamma = \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r}$$

$$\text{s.t} \quad \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1}x_{ij} \le \theta_i \sum_{j=1}^{n}x_{ij} \qquad \forall i$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1}z_{gj} \ge \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{2}z_{gj} \qquad \forall g$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{2}y_{kj} \ge \varphi_r \sum_{j=1}^{n}y_{kj} \qquad \forall k$$

$$\sum_{j=1}^{n}\lambda_{jr}^{1} = 1 \quad \forall r \qquad (6)$$

$$\sum_{j=1}^{n}\lambda_{jr}^{2} = 1 \quad \forall r$$

$$\lambda_{jr}^{1} \ge 0 \qquad \forall r,j$$

$$\lambda_{jr}^{2} \ge 0 \qquad \forall r,j$$

$$\theta_i \le 1 \qquad \forall i$$

$$\varphi_r \ge 1 \qquad \forall r$$

Here, θ_i represent the contraction variable associate with the ith input for $i = 1, \dots, m$ and ϕ_r shows the expanding factor for the rth output; $r = 1, \dots, s$. The optimal value of the objective function in this model is $0 < \gamma^* \le 1$ and the Pareto efficient operating point for any observed units could be introduced based on the optimal solution of the model (6).

Note that the first constraint relates to decreasing the sum of inputs, while the second and third constraints relate to intermediate products and increasing the total output, respectively. Unlike model (3), model (6) assumes the presence of intermediate products, resulting in decreasing the inputs and increasing the outputs in one phase, in a non-radial approach.

By solving the model (6), the corresponding vector $(\lambda_{1r}^{1*}, \lambda_{2r}^{1*}, \dots, \lambda_{nr}^{1*}, \lambda_{1r}^{2*}, \lambda_{2r}^{2*}, \dots, \lambda_{nr}^{2*})$ is defines for each DMU$_r$ the operating point at which it should aim. The inputs, intermediate products and outputs of each such point in the model (3) can be computed as follows:

$\hat{x}_{ir} = \sum_{j=1}^{n} \lambda_{jr}^{1*} x_{ij}$ (i=1,...,m), $z_{gj}^{out} = \sum_{j=1}^{n} \lambda_{jr}^{1*} z_{gj}$ (g=1,...,h), $z_{gj}^{in} = \sum_{j=1}^{n} \lambda_{jr}^{2*} z_{gj}$ (g=1,...,h),

and $\hat{y}_{kr} = \sum_{j=1}^{n} \lambda_{jr}^{2*} y_{kj}$ $(k = 1, \ldots, s)$.

Theorem 1. The objective value obtained by the model (6) is less than or equal to that of the model (6).

Proof. By contradiction, let $(\bar{\lambda}, \bar{\theta})$ and $(\tilde{\lambda}^{1}, \tilde{\lambda}^{2}, \tilde{\theta}, \tilde{\varphi})$ be the optimal solution of model (3) and model (6), respectively. Without loss of generality, assume that the sum of one of the inputs or outputs (or both) is in a better condition and is considered as the input, then we show that $\bar{\theta}_t > \tilde{\theta}_t$, and we have $\bar{\theta}_t = \frac{\sum_{r=1}^{n} \sum_{j=1}^{n} \bar{\lambda}_{jr} x_{tj}}{\sum_{j=1}^{n} x_{tj}} = s$, then $\tilde{\theta}_t < s$ and $\frac{\tilde{\theta}_t}{s} < 1$, which in this case is a better value than the $\tilde{\theta}_t$, which is a contradiction.

By summing the total inputs of the system and expanding the total outputs of the system, this model seeks to reduce the total consumed inputs and increase the sum of produced outputs, and then, while considering intermediate products into account.

3.1. Linearization issue

The provided non-linear optimization model (6) is transformed into a linear programming equivalence through a well–known treatment of 'fractional programming' (Charnes and Cooper (1962). To briefly review treatment applied to model (6), a new variable $\beta = (\sum_{r=1}^{s} \varphi_r / s)^{-1}$ is included in the model (6). Here, the variable satisfies both conditions $0 \le \beta \le 1$ and $\beta = (\sum_{r=1}^{s} \varphi_r / s)^{-1} = 1$. Then, all variables in the model (6) can be transformed as follows: $\theta_i' = \beta\theta_i$ (i=1,...,m), $\varphi_r' = \beta\varphi_r$ (r=1,...,s), $\lambda_{jr}^{1'} = \beta\lambda_{jr}$ and $\lambda_{jr}^{2'} = \beta\lambda_{jr}$, (j,r=1,...,n).

By using these transformed variables, model (6) can be reformulated as the following linear programming model:

$$
\begin{aligned}
\text{Min} \quad & \frac{1}{m}\sum_{i=1}^{m}\theta_i' \\
& \frac{1}{s}\sum_{i=1}^{m}\varphi_r' = 1 \\
\text{s.t} \quad & \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1'}x_{ij} \le \theta_i'\sum_{j=1}^{n}x_{ij} \qquad \forall i \\
& \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{1'}z_{gj} \ge \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{2'}z_{gj} \qquad \forall g \\
& \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{2'}y_{kj} \ge \varphi_r'\sum_{j=1}^{n}y_{kj} \qquad \forall k \\
& \sum_{j=1}^{n}\lambda_{jr}^{1'} = \beta \quad \forall r \\
& \sum_{j=1}^{n}\lambda_{jr}^{2'} = \beta \quad \forall r \\
& \theta_i' \le \beta,\ \varphi_r' \ge \beta \\
& \lambda_{jr}^{1'} \ge 0 \qquad \forall r,j \\
& \lambda_{jr}^{2'} \ge 0 \qquad \forall r,j
\end{aligned}
\tag{7}
$$

Model (7) is structured under variable return to scale technology, depending on β value. The basic form of model (7) is described in Azadi et al. (2012). The dual formulation of this model takes the following linear form:

$$\begin{aligned}
\max \quad & \sum_{k=1}^{s}\sum_{j=1}^{n} u_k y_{kj} + \sum_{r=1}^{n} \xi_r - \sum_{i=1}^{m}\sum_{j=1}^{n} v_i x_{ij} \\
& \sum_{i=1}^{m}\sum_{j=1}^{n} v_i x_{ij} \geq \frac{1}{m} \\
& \sum_{k=1}^{s}\sum_{j=1}^{n} u_k y_{kj} \geq \frac{1}{s}\left(1 - \sum_{i=1}^{m}\sum_{j=1}^{n} v_i x_{ij} + \sum_{k=1}^{s}\sum_{j=1}^{n} u_k y_{kj} + \sum_{r=1}^{n} \xi_r\right) \\
& \sum_{k=1}^{s}\sum_{j=1}^{n} u_k y_{kj} - \sum_{i=1}^{m}\sum_{j=1}^{n} v_i x_{ij} + \sum_{r=1}^{n} \xi_r \leq 0 \\
s.t \quad & \sum_{g=1}^{h}\sum_{j=1}^{n} w_g z_{gj} - \sum_{i=1}^{m}\sum_{j=1}^{n} v_i x_{ij} \leq 0 \\
& \sum_{k=1}^{s}\sum_{j=1}^{n} u_k y_{kj} - \sum_{g=1}^{h}\sum_{j=1}^{n} w_g z_{gj} \leq 0 \\
& v_i \geq 0, \quad \forall i=1,....m \\
& u_k \geq 0, \quad \forall k=1,...,s \\
& w_g \geq 0, \quad \forall g=1,...,h \\
& \xi_r \quad free
\end{aligned} \tag{8}$$

In the model (8), separate shadow prices are available for each input, output, and intermediate product, aiming to create a balance in the intermediate products. The difference between model (8) and classical models is that the performance of each DMU is evaluated separately and the profit of each unit of DMU is presented as a limitation in the dual model. However, in the model (8), the performance is evaluated on the total system of production and profit and the entire DMUs are considered in constraints of the model.

4. An illustrative example

This section compares a numerical example with six DMUs including one input, one output, and one intermediate product with the numerical results obtained from the models (3) and (6), which is presented in Table 1. Figures 1 and 2 provide a graphical interpretation for the results of the model (3) and the proposed model (6), respectively.

	Data			MODEL (3)		MODEL (6)			
DMU	x	z	y	x^*	y^*	x^*	z^{*out}	z^{*in}	y^*
A	2	8	8	2	8	1	3	2	10
B	1	3	2	1.56	6.25	1	3	2	10

C	1.5	4	6	1.56	6.25	1	3	2	10
D	3	2	10	2	8	1	3	2	10
E	2	1	7	1.56	6.25	1	3	2	10
F	4	8	8	1.56	6.25	1	3	2	10
Total	13.5	26	41	10.25	41	6	18	12	60
θ^*				0.7592593			0.444444		
φ^*				---			1.463415		

Table 1. Data and targets of six DMUs

To properly describe model (6), it is rewritten to evaluate the graph efficiency measure for total inputs and outputs given in the illustrative example as follows:

$$\min \theta/\varphi$$

s.t

$$2 \lambda^1_{11} + 1 \lambda^1_{12} + 1.5 \lambda^1_{13} + 3 \lambda^1_{14} + 2 \lambda^1_{15} + 4\lambda^1_{16} + 2 \lambda^1_{21} + 1 \lambda^1_{22} + 1.5 \lambda^1_{23} + 3 \lambda^1_{24} + 2 \lambda^1_{25} + 4\lambda^1_{26} + 2 \lambda^1_{31} + 1 \lambda^1_{32} + 1.5 \lambda^1_{33} + 3 \lambda^1_{34} + 2 \lambda^1_{35} + 4\lambda^1_{36} + 2 \lambda^1_{41} + 1 \lambda^1_{42} + 1.5 \lambda^1_{43} + 3 \lambda^1_{44} + 2 \lambda^1_{45} + 4\lambda^1_{46} + 2 \lambda^1_{51} + 1 \lambda^1_{52} + 1.5 \lambda^1_{53} + 3 \lambda^1_{54} + 2 \lambda^1_{55} + 4\lambda^1_{56} + 2 \lambda^1_{61} + 1 \lambda^1_{62} + 1.5 \lambda^1_{63} + 3 \lambda^1_{64} + 2 \lambda^1_{65} + 4\lambda^1_{66} \geq 13.5$$

$$8 \lambda^1_{11} + 3 \lambda^1_{12} + 4 \lambda^1_{13} + 2 \lambda^1_{14} + 1 \lambda^1_{15} + 8\lambda^1_{16} + 8 \lambda^1_{21} + 3 \lambda^1_{22} + 4 \lambda^1_{23} + 2 \lambda^1_{24} + 1 \lambda^1_{25} + 8\lambda^1_{26} + 8\lambda^1_{31} + 3 \lambda^1_{32} + 4 \lambda^1_{33} + 2 \lambda^1_{34} + 1 \lambda^1_{35} + 8\lambda^1_{36} + 8\lambda^1_{41} + 3 \lambda^1_{42} + 4 \lambda^1_{43} + 2 \lambda^1_{44} + 1 \lambda^1_{45} + 8\lambda^1_{46} + 8\lambda^1_{51} + 3 \lambda^1_{52} + 4 \lambda^1_{53} + 2 \lambda^1_{54} + 1 \lambda^1_{55} + 8\lambda^1_{56} + 8\lambda^1_{61} + 3 \lambda^1_{62} + 4 \lambda^1_{63} + 2 \lambda^1_{64} + 1 \lambda^1_{65} + 8\lambda^1_{66} \geq 8 \lambda^2_{11} + 3 \lambda^2_{12} + 4 \lambda^2_{13} + 2 \lambda^2_{14} + 1 \lambda^2_{15} + 8\lambda^2_{16} + 8 \lambda^2_{21} + 3 \lambda^2_{22} + 4 \lambda^2_{23} + 2 \lambda^2_{24} + 1 \lambda^2_{25} + 8\lambda^2_{26} + 8\lambda^2_{31} + 3 \lambda^2_{32} + 4 \lambda^2_{33} + 2 \lambda^2_{34} + 1 \lambda^2_{35} + 8\lambda^2_{36} + 8\lambda^2_{41} + 3 \lambda^2_{42} + 4 \lambda^2_{43} + 2 \lambda^2_{44} + 1 \lambda^2_{45} + 8\lambda^2_{46} + 8\lambda^2_{51} + 3 \lambda^2_{52} + 4 \lambda^2_{53} + 2 \lambda^2_{54} + 1 \lambda^2_{55} + 8\lambda^2_{56} + 8\lambda^2_{61} + 3 \lambda^2_{62} + 4 \lambda^2_{63} + 2 \lambda^2_{64} + 1 \lambda^2_{65} + 8\lambda^2_{66}$$

$$8 \lambda^2_{11} + 2 \lambda^2_{12} + 6 \lambda^2_{13} + 10 \lambda^2_{14} + 7 \lambda^2_{15} + 8\lambda^2_{16} + 8 \lambda^2_{21} + 2 \lambda^2_{22} + 6 \lambda^2_{23} + 10 \lambda^2_{24} + 7 \lambda^2_{25} + 8\lambda^2_{26} + 8\lambda^2_{31} + 2 \lambda^2_{32} + 6 \lambda^2_{33} + 10 \lambda^2_{34} + 7 \lambda^2_{35} + 8\lambda^2_{36} + 8\lambda^2_{41} + 2 \lambda^2_{42} + 6 \lambda^2_{43} + 10 \lambda^2_{44} + 7 \lambda^2_{45} + 8\lambda^2_{46} + 8\lambda^2_{51} + 2 \lambda^2_{52} + 6 \lambda^2_{53} + 10 \lambda^2_{54} + 7 \lambda^2_{55} + 8\lambda^2_{56} + 8\lambda^2_{61} + 2 \lambda^2_{62} + 6 \lambda^2_{63} + 10 \lambda^2_{64} + 7 \lambda^2_{65} + 8\lambda^2_{66} \leq 41$$

$$\lambda^1_{11} + \lambda^1_{12} + \lambda^1_{13} + \lambda^1_{14} + \lambda^1_{15} + \lambda^1_{16} = 1,\ \lambda^1_{21} + \lambda^1_{22} + \lambda^1_{23} + \lambda^1_{24} + \lambda^1_{25} + \lambda^1_{26} = 1,\ \lambda^1_{31} + \lambda^1_{32} + \lambda^1_{33} + \lambda^1_{34} + \lambda^1_{35} + \lambda^1_{36} = 1,\ \lambda^1_{41} + \lambda^1_{42} + \lambda^1_{43} + \lambda^1_{44} + \lambda^1_{45} + \lambda^1_{46} = 1,\ \lambda^1_{51} + \lambda^1_{52} + \lambda^1_{53} + \lambda^1_{54} + \lambda^1_{55} + \lambda^1_{56} = 1,\ \lambda^1_{61} + \lambda^1_{62} + \lambda^1_{63} + \lambda^1_{64} + \lambda^1_{65} + \lambda^1_{66} = 1,\ \lambda^2_{11} + \lambda^2_{12} + \lambda^2_{13} + \lambda^2_{14} + \lambda^2_{15} + \lambda^2_{16} = 1,\ \lambda^2_{21} + \lambda^2_{22} + \lambda^2_{23} + \lambda^2_{24} + \lambda^2_{25} + \lambda^2_{26} = 1,\ \lambda^2_{31} + \lambda^2_{32} + \lambda^2_{33} + \lambda^2_{34}$$

$+\lambda^2_{35} +\lambda^2_{36} =1,\ \lambda^2_{41} +\lambda^2_{42} +\lambda^2_{43} +\lambda^2_{44} +\lambda^2_{45} +\lambda^2_{46} =1,\ \lambda^2_{51} +\lambda^2_{52} +\lambda^2_{53} +\lambda^2_{54} +\lambda^2_{55} +\lambda^2_{56} =1,\ \lambda^2_{61}+\lambda^2_{62} +\lambda^2_{63} +\lambda^2_{64} +\lambda^2_{65} +\lambda^2_{66}=1.$

According to Table 1, applying model (6) results in a decrease in total inputs by 7.5 units and an increase in total output by 19 units. Compared to the model (3), there is a further decrease in input by 4.25 units and an increase in output by 19 units. Figures 2 and 3 show the solution and projection points in models 3 and 6.

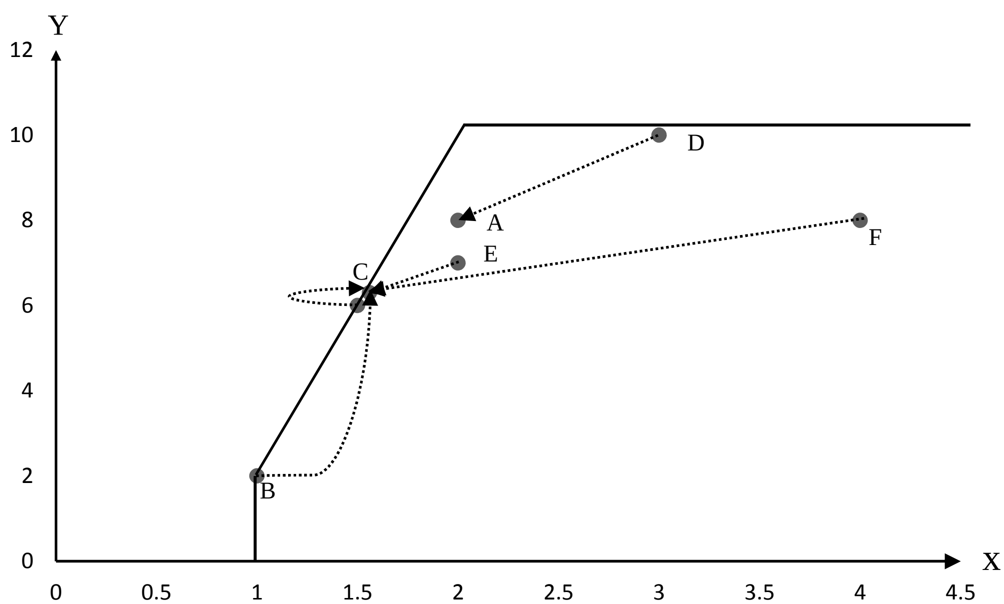

Figure 2. Production possibility set and project point in black box production

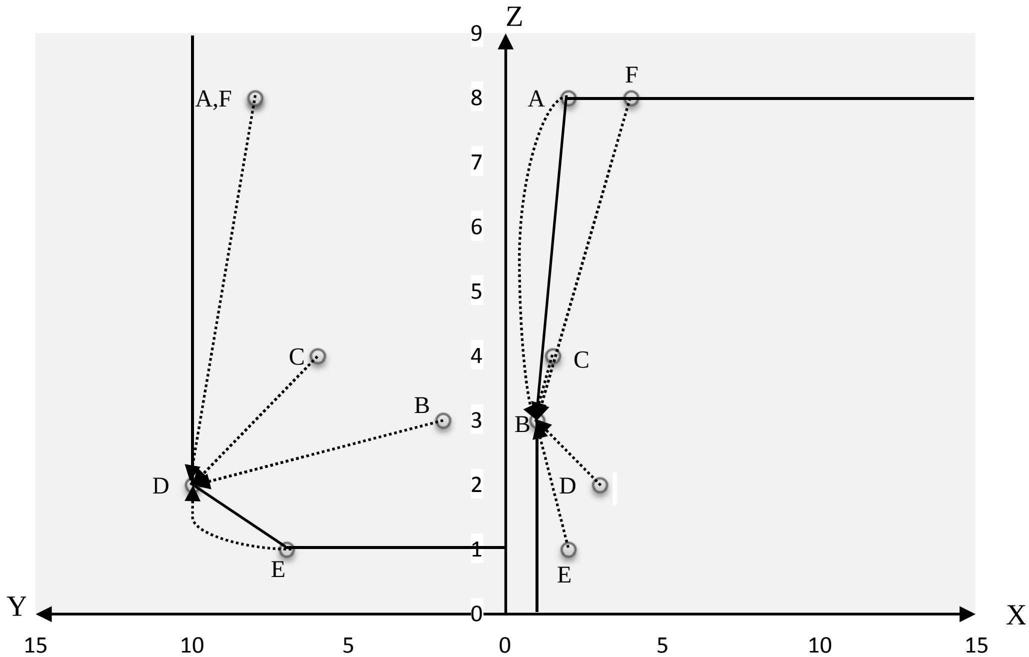

Figure 3. Production possibility set and project points in two-stage case

As shown in Figure 2, points B, C, E, and F are projected on the (1.56,6.25) near the efficient frontier while point B is projected onto point A, which is inefficient. Model (3) requires a second phase for maximizing slack variables to visualize the points on to the efficient boundary.

Note that the new proposed approach projects as many DMUs as possible onto technically efficient points operating at their most productive scale size (MPSS). In other words, the model detects a few efficient units operating at their MPSS and finds that the maximum intra-organizational efficiency is attained by replicating them as much as possible.

In the first stage (Figure 3), points A and B are on the technically efficient points and point F is on the weak efficient frontier, although points C, D, and E are inefficient. After improving inefficient points, efficient and inefficient points are represented on the optimal point B. In this case, the situation of inefficient points will improve, and the efficient points will remain efficient, resulting in minimizing the number of image points. In the second stage, points D and E are on the technically efficient, and points A, B, C, and F are inefficient; therefore, inefficient points are improved to the point D, and the efficient points are depicted on the efficient d and remain efficient. As observed, the image points below the first and second sections are not necessarily the same because each sub-process is examined separately.

5. Empirical Application

Service industries generally serve to their customers for profit. So far, a large body of research has been conducted to evaluate performance and banking. In this regard, numerous studies have used the DEA technique to measure the management performance of the industry. (Sherman and Gold 1985) used DEA to measure the performance of banking systems for the first time. Since then, the application of the DEA has made great achievements in assessing the application of the banking industry.

In this section, we examine the performance of eight Chinese commercial banks, the data of which are summarized in Table 2. The data were collected from the official website of each bank in Zhu et al. (2018). Totally, four banks were state-owned commercial banks, namely the Bank of China (BOC), the Agricultural Bank of China (ABC), the China Construction Bank (CCB), and the Industrial and Commercial Bank of China (ICBC). Other banks, Nanning Bank (NB), Shanghai Pudong Development Bank (SPDB), Chongqing Rural Commercial Bank (CQRCB) and Ningbo Bank (NBCB), were joint-stock banks. Figure 4 demonstrates the network production system of the banks. Table 3 reports the new model solutions for the case study.

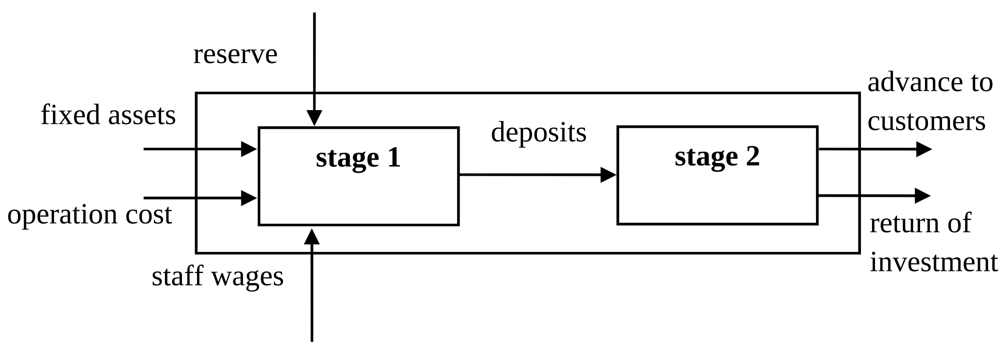

Figure 4. Structure of bank system

The inputs of the system, which are also the inputs of the first stage, are:

- x_1: Fixed assets
- x_2: Operation cost
- x_3: Staff wages
- x_4: Reserve

The outputs of the system, which are also the outputs of the second stage, are:

- y_1: Advances to customers
- y_2: Return to investment

There are also intermediate products in the system, which are the output of the first stage and the input of the second stage of the production system:

- z: Deposits

Table 2. Total number of inputs, intermediate product, and outputs for the problem (Zhu et al. 2018)

	x_1	x_2	x_3	x_4	z	y_1	y_2
Total	556464.63	908299.07	145414.16	507808.86	47873027.56	30697566.71	5237075.70

Table 3. Solution of model (6)

Bank	x_1^*	x_2^*	x_3^*	x_4^*	z^{*out}	z^{*in}	y_1^*	y_2^*
ABC	88830.46	168878.42	26542.12	66592.48	9115896.26	3091970.73	2255245.92	894308.98
NB	10101	38533	7214	23050	2134365	8480932.16	6202404.36	894308.98
BOC	10101	38533	7214	23050	2134365	8480932.16	6202404.36	163755.37
CCB	88830.46	168878.42	26542.37	66592.48	9115896.26	207577.27	142564.62	1163755.37
ICBC	81071.84	156033.15	24637.37	62301.46	8427881.01	207577.27	142564.62	1163755.37
SPDB	62906.46	125958.34	20177.76	52254.83	6817020.93	848093216	6202404.36	1163755.37
CQRCB	10101	38533	7214	23050	2134365	4583302.09	3347575.35	1138946.69
NBCB	10101	38533	7214	23050	2134365	8480932.16	6202404.36	990832.07
Total	362043.22	773880.33	126755.37	339941.25	42014156	42014156	30697566.71	573417.61
θ_i*	0.65	0.85	0.87	0.67	–	–	–	–
φ_r*	–	–	–	–	–	–	1	1.64

According to the results of Tables 3 and 4, the total fixed assets (x_1) was initially equal to 556464.63. After the implementation of the new model, this input was decreased to 362043.22 (θ_1 =0.65), which shows about 35% saving in a total amount of this input. The total operation costs (x_2) were initially equal to 908299.07; by implementing the model, this input was decreased to a total of 773880.33 (θ_2 =.85), which also shows about 15% saving in the total amount of this input. For the third input, the total staff wages (x_3) was first equal to 145414.16 and is decreased to 126755.37 (θ_3 =.87) after the model implementation. This input shows about 13% saving. Finally, the total reserve (x_4) was initially equal to 507808.86, which is decreased to 339941.25 (θ_4 =.67) after the model implementation; this input shows about 34% saving. Given the constraints of the model (4), intermediate product (z) that is generated in the first stage, is then used in the second stage. The results indicates that total of the advances to customers (y_1) remains unchanged after the implementation of the new model (φ_1 =1). The total return to investment (y_2) was initially equal to 5237075.7, which was increased to 8573417.61 (φ_2 =1.64) with a 63 % increase in this output.

Based on the provided results, the central decision-maker can obtain useful information on the inputs excesses and outputs shortfalls and the associated sources of system inefficiencies in performance analysis of two-stage production systems. As the results, efficient benchmark targets are computed which provides improvement path for inefficient DMUs.

6. Conclusion

Conventional DEA models cannot be applied to centralized resource allocation in production systems with network structure. They also fail to provide correct information on these production systems because of the ignorance of the relationships between the manufacturing sub-sectors. This paper proposed a new network DEA model, in which all units were categorized under the supervision of a centralized DM, which not only wants them to be efficient but is also concerned about total input consumption and total output production. Conventional network DEA models set targets separately for each DMU; a different approach that projects all the units simultaneously are needed to project all DMUs onto the efficient frontier.
This article provided a brief review of some basic network systems with their associated models. In real-world applications, some of the network production systems are two-stage simple series structure. In this study, a new two-stage network DEA model was proposed which can be used for centralized resource allocation of the two-stage network structures. Moreover, the method improves the inefficient units in their projection on efficient frontier. Some numerical examples were then used to illustrate the approach. Development of models for performance analysis of general network systems to identify sub-processes and total system relation are interesting challenges for future studies.

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